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A Review of Previous Studies of Oscillatory Combustion in Gas Turbines:

Progress Report-I Funded by Solar Turbines Incorporated

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Motivation

Combustion-generated pollutants, in particular NO_X and carbonaceous soot, may be significantly reduced by use of lean-burning combustion systems in gas-turbine engines. However, as the fuel-air mixture becomes lean, small perturbations in the flow can change the unsteady heat-release pattern, and if the heat release is in-phase with the acoustic pressure fluctuations in the combustor, the fluctuations can grow into high-amplitude oscillations, typically at frequencies in the 100-500 Hz range. These oscillations can cause flame extinction, reduce engine life and/or cause catastrophic structural damage under extreme circumstances. Only to the extent that such oscillations can be controlled and lean mixtures can be burned stably, can emissions be further reduced to meet future standards [1].

Combustion Instabilities

Combustion instabilities arise at different stages of the combustion process and could be grouped into various types [2]. Intrinsic instabilities are inherent to the combustion and fluid physics. Chamber instabilities result from the interaction of the combustion process with a combustion chamber. System instabilities involve the interaction of the combustion processes in a chamber with upstream feed lines and/or downstream exhaust. In each of these three categories, different physical processes may contribute to the instability. Another categorization of instabilities is in terms of the physical processes involved. For example, there are buoyant instabilities, hydrodynamic instabilities, and acoustic instabilities, among others. The focus of this research is on oscillatory types of combustion instabilities that are acoustic chamber instabilities, sometimes with feed-line coupling and with influence of hydrodynamic and intrinsic instability.

Characteristics of Oscillatory Instabilities

The oscillatory instability in gas-turbine combustors arises from the coupling of unsteady heat release with acoustic waves in a chamber, resulting in repeated pressure fluctuations at various characteristic frequencies. The instability frequencies are associated with the geometry of the device and may be influenced by interactions between the device and the flow field. The interactions causing these self-excited oscillations are complex because of the coupling of the flow field with the unsteady (and highly nonlinear) heat release. Some experiments have been interpreted [3 ,4] to indicate that a primary cause for generation of instabilities is an acoustic wave generated by unsteady heat release that trips a Kelvin-Helmholtz instability in the flow [5], where high density gradients, shear, and substantial vorticity exist. The instability modifies both the overall flame structure and the flow (turbulence), and hence an effective closed-loop feedback system is generated.

In 1878 Rayleigh proposed a criterion that has evolved into a clear rule for the potential amplification of an acoustic wave in a combustion system, essentially that positive correlation of the heat-release and pressure variations over the period of one acoustic cycle results in amplification of oscillations [6],

$$\int_0^T p'(t)q'(t)dt > 0\,, \tag{1}$$

where p' and q' are the pressure and the heat-release perturbations, respectively, as a function of time *t*, and *T* denotes the period of the oscillation. Unfortunately, it is difficult to apply this criteria in a practical setting, as may be observed in studies [7-9] of one-dimensional systems designed to model reheat buzz. For linear oscillatory instability studies, the perturbations of concern can be represented as harmonic oscillations, the real (Re) part of complex functions,

$$p' = \mathrm{Re}\{Pe^{(i\omega t-\alpha)}\} \quad \text{and} \quad q' = \mathrm{Re}\{Qe^{(i\omega t-\beta)}\}, \tag{2}$$

where P and Q are amplitudes of the pressure and heat-release perturbations, respectively, and ω is the frequency. In terms of α and β, the phase angles of the pressure and heat-release perturbations, respectively, it can be shown that equation (1) is satisfied if the phase difference between the two perturbations, $\delta = \alpha - \beta$, lies between 0 and π.

To gain further understanding of the oscillatory instability from first principles, an equation for the acoustic energy may be derived from fundamental fluid mechanics. Starting from the continuity, momentum and energy relations, and considering only linear perturbations, with the brackets denoting an average over the cycle, this equation takes the form

$$\frac{\partial}{\partial t}\int_V \left(\frac{1}{2}\rho <u'^2> + \frac{1}{2}\frac{<p'^2>}{\bar{\rho}\bar{c}^2} \right) dV + \int_S \left(<p'u'> - <u'\tau'> \right) dS = \int_V \frac{\gamma - 1}{\bar{\rho}\bar{c}^2} <p'q'> dV - \int_V <\frac{\partial u'}{\partial x}\tau'> dV, \tag{3}$$

where u', ρ', τ', p' and q' are the perturbations for the velocity, density, viscous stress, pressure and heat-release rate respectively. Here dS and dV are the differential surface-area and volume elements respectively, $\bar{\rho}$ is the average density, and $\bar{c}$ is the average speed of sound, given by:

$$\bar{c} = \sqrt{\gamma R \theta}, \tag{4}$$

where γ, R and θ are the ratio of specific heats, the gas constant and the temperature, respectively. For simplicity of notation, tensor contractions are not shown in equation (3), terms being written in the form they would take in a one-dimensional system.

It is worth noting that:

- The first term represents the rate of change in the acoustic energy in the combustor volume.
- The second term is a convection term for energy moving in and out of the control volume through the surface S.
- The third term arises from the coupling between the pressure and heat perturbations (essentially the Rayleigh criteria).
- The fourth term represents the viscous dissipation of the acoustic energy.

The thermodynamic interpretation of the Rayleigh criteria may also be illustrated in terms of the mechanical work done over the period of one cycle by the acoustic energy [10],

$$\oint p' d\upsilon' = -\frac{\upsilon}{\gamma p}\oint p' dp' + \oint p' d\upsilon'^{(q)} = 0 + \int_0^T p' \frac{d\upsilon'^{(q)}}{dt} dt \sim \int_0^T p' \dot{q}' dt\,, \qquad (5)$$

where p' and υ' are the perturbations in pressure and specific volume, respectively. This work term is split into an isentropic portion, the integral of which vanishes, and a portion, ($\upsilon^{(q)}$), resulting from the volume change caused by the heat addition. Depending on the sign of the integral, mechanical work could be added to or extracted from the cycle.

Amplification of Oscillatory Combustion Instability

Several mechanisms have been identified that contribute to acoustic instability amplification, some of which are:

Air/fuel-ratio fluctuations: Recently investigated by Lieuwen and Cho [11], the heat- release perturbations at the flame can cause acoustic waves to propagate upstream into the feed lines and cause perturbations in the incoming air/fuel mixture. These perturbations may be carried by the mean flow and trigger a fluctuation at the base of the flame, closing the instability loop. Several studies have addressed this possible mechanism. For example, Sacarini et al.[12] studied the fuel-air fluctuations in a simple duct and concluded that the strong potential of these fluctuations to drive instabilities justified substantial effort in mitigating air/fuel fluctuations. In their work [13], they defined a parameter σ as an indicator of the efficiency of the mixing duct,

$$\sigma = {\frac{\phi'}{\phi}} \Big/ {\frac{u'}{u}}, \qquad (6)$$

where $\frac{\phi'}{\phi}$ is the ratio of air/fuel-ratio perturbations to the mean air/fuel-ratio, and $\frac{u'}{u}$ is the ratio of velocity perturbations to the mean flow velocity.

Their later work focused on trying to get this parameter as close to zero as possible [13], which was accomplished by improving the mixing quality of the reactants by using multiple fuel-injection locations.

Convective-Acoustic Waves: This is a class of perturbations that are carried by the mean flow, such as vortices shed from the flame holder and/or entropy waves propagating downstream, and generating upstream-propagating acoustic waves. The vortex shedding and entropy phenomena are as follows:

Vortex Shedding: Vortices shed from the flame holder were suggested as a cause of combustion instabilities as early as 1956 by Rogers and Marble [14]. More recently, experimental investigation by Poinsot et al. [3] looked at this as a possible source of combustion instability. The instability is triggered when the vortices shed at the flame holder entrain unburned mixture, which propagate downstream and causes a sudden heat release at some point downstream. This triggers an acoustic wave propagating upstream, closing the feedback loop. Culick and Magiawalla [15] investigated vortices shed from the flame holder consisting of pure products, which would impinge on obstacles downstream (e.g. the nozzle) and cause pressure oscillations to intensify. This investigation was of purely acoustic phenomenon with no heat-release contribution. Mateev and Culick [16] recently investigated the formation of vortices behind flame holders and their interaction with flow-field perturbations in premixed combustors, using a newly developed quasi-steady model. In this model, they addressed a dump combustor in which they assumed constant fluid properties, vortex burning as the only source for instability (without vortex-surface interaction), and vortex propagation at the mean flow velocity. They were able to partially validate the model against experimental results for vortex shedding in a non-reacting oscillating cold flow. The authors cited a concern that reacting flows might behave differently and that resulting vortex shedding and interaction in reacting flows might follow a different pattern [16].

Entropy waves: The phenomenon of localized hot spots in a mean gas flow has been known for a number of years. When these hot spots reach the inlet of a chocked nozzle, their arrival triggers an upstream acoustic wave propagation that can cause an acoustic instability. The effect of entropy waves on flow-field instabilities was suggested in early work by Chu [17], who considered their influence on combustion instabilities to be minimal except at low frequencies. Polifke et al. [18] recently studied the constructive or destructive coupling of entropy waves with the pressure perturbations. Since the hot spots are transported by the mean flow (usually at low velocity), effects of entropy waves have been assumed to exist at low frequencies [19].

There are other possible sources of combustion oscillatory instability ranging from purely chemical-kinetic to solely fluid-mechanical. Their contributions vary with oscillation modes. It is also possible that some of the oscillation modes are triggered by a combination of perturbations (velocity, temperature, laminar flame speed, etc...).

Damping of Oscillatory Combustion Instability

Little work has been done to include damping effects in the body of literature on combustion instabilities. Most of the previous research has looked at amplification mechanisms and signs of growth or decay of the modes. The few studies looking into damping were independent from research into amplification or were complete engine studies [20].

Williams [2] presented some of the possible damping sources in an oscillating combustor environment.

Wall damping: Wall damping of the velocity parallel to the wall occurs as a result of the oscillating boundary layer with a thickness of order

$$\delta = \sqrt{\frac{\mu_w}{\rho_w \omega}}, \qquad (7)$$

where μ_w is the viscosity near wall. Here ω and ρ_w are the oscillatory frequency and the density near the wall, respectively. The wall damping depends on gas molecular properties near the wall and on the surface area of the combustor liner and wall. This energy damping can be found, starting from a simplified time-dependent momentum equation [2],

$$\rho_w \frac{\partial v}{\partial t} = \frac{\partial\left(\mu_w \frac{\partial v}{\partial y}\right)}{\partial y}, \tag{8}$$

where ρ_w, μ_w and v are the density, viscosity and velocity values near the wall. This equation has a solution,

$$v = \left(\frac{\theta_w}{\theta_c}\right) \mathrm{Re}\left\{V e^{-(1+i)\sqrt{\frac{\omega\rho_w}{2\mu_w}}y}\right\}, \tag{9}$$

where θ_w, θ_c, V and, ω are the temperature at the wall, the temperature at the core, the oscillatory velocity magnitude outside the oscillating boundary layer, and the frequency of the oscillation respectively.

From this velocity solution, an energy dissipation expression, (per unit area of combustor wall), was formulated by Williams [2] as,

$$e_{wd} = \int_0^\infty \frac{1}{2}\rho_w \omega \left[\frac{\theta_w}{\theta_c} V.e^{-(1+i)\sqrt{\frac{\omega\rho_w}{2\mu_w}}y}\right]^2 dy = \frac{1}{2}\left(\frac{\theta_w}{\theta_c}\right)^2 |V|^2 \sqrt{\frac{\omega\rho_w\mu_w}{2}}, \tag{10}$$

This solution then needs to be integrated over the wall boundary surfaces to obtain the total rate of dissipation by the wall damping.

Particle damping: This is the sound attenuation by the Stokes drag on small particles (mainly soot in the combustion applications of interest here). Williams [2] identified the range of contributing particle sizes by

$$r_s << \sqrt{\frac{\mu}{\rho\omega}} << \lambda, \tag{11}$$

where r_s, μ, ρ, ω ,and λ are the mean particle radius, viscosity in the chamber, density, frequency, and the acoustic wave length. The contribution of the particle damping depends on the velocity difference between particles and gas, particle sizes, particle density, number of particles per unit volume and particles distribution.

The total rate of acoustic energy dissipated by solid, spherical particles in the flow can be calculated as

$$e_{pd} = \frac{1}{2}\int_V \frac{1}{\tau}\left\langle n_s\left(\frac{4}{3}\pi r_s^{\,3}\rho_s\right)\left(v_g - v_s\right)\cdot v_g\right\rangle dV \,, \tag{12}$$

where n_s, r_s, ρ_s, v_g ,and v_s are the number of particles, particle radius, particle density, gas velocity and particle velocity, and the brackets represent an average over time. The response time, τ , is calculated from

$$\tau = \frac{2 r_s^{\,2}\rho_s}{(9\mu)} \,, \tag{13}$$

where μ is the gas viscosity.

Equating the force decelerating a particle with the Stokes drag and performing a few algebraic operations reduces the coefficient of energy dissipation to

$$\alpha_{pd} = \left(\frac{\omega}{2}\right)\left(\frac{\rho_s}{\rho_g}\right)\int_0^{\infty}\left\{\frac{4}{3}\pi r_s^{\,3}(\omega\tau)\Big/\left[1 + (\omega\tau)^2\right]\right\}G(r_s)\,dr_s \,, \tag{14}$$

where $G(r_s)\,dr_s$ represents the number of particles per unit volume within radius range dr_s about r_s.

The total energy dissipated can be calculated from

$$e_{pd} = e_o \exp\left(-\alpha_{pd} t\right) \tag{15}$$

where e_o is the initial acoustic energy.

Relaxation damping: This is a form of homogeneous dissipation that is caused by chemical and vibration relaxation. The contribution of relaxation damping depends on the frequency of the oscillations, the chemical relaxation time, which is a measure of the reaction rate, and the speed-of-sound ratio, $\frac{a_f}{a_e}$, where a_f and a_e are the frozen and equilibrium sound speeds respectively. Relaxation damping is usually small compared with wall damping, but might become important for large chambers.

Homogeneous damping: This is damping of the acoustic energy caused by viscous dissipation. It primarily depends on velocity gradients, acoustic frequency, and viscous properties. It is believed to have small contribution to acoustic energy attenuation at low frequencies, but this contribution increases as frequencies increase, and it is a possible cause for the absence of very high-frequencies.

Nozzle damping: When choked flow conditions are met, the propagation of acoustic waves is restricted at the nozzle and no propagation of longitudinal waves is allowed upstream from a diverging supersonic-flow section. This boundary can absorb energy depending on the flow conditions.

There are other damping sources to be considered in an actual combustor, such as perforated walls and cooling flows near the wall, and heat-addition mechanisms acting out of phase with the pressure perturbations.

Review of Prior Methods to Study Oscillatory Instability

There is a large body of literature in the area of combustion instabilities. Before instabilities were recognized as problems in the gas-turbine industry, combustion instability was observed as a serious issue during the development of solid rockets, and later also became pronounced for liquid rockets, where much of the earlier research was conducted [21]. The complicated nature of the coupling between the heat release and acoustics continues to make control of instabilities a challenging task for researchers looking into investigating and designing propulsion and power-generation devices.

Figure 2 summarizes some of the methods used to analyze and/or predict the onset of combustion instabilities. All approaches to study combustion oscillations start from the Navier-Stokes conservation equations, and may be classified as follows:

- An approach in which the linearized Rankine-Hugoniot equations are expanded around a small Mach number to relate the perturbations across a flame front. The use of network models and the flame jump relations produces a dispersion equation to be solved for the eigenfrequencies of interest.

- Another approach by which simplifying the linearized (N-S) equations produces an inhomogeneous wave equation for pressure perturbations with a source term accounting for combustion heat release. The challenge in this approach becomes finding approximate solutions that satisfy the boundary conditions. The method of weighted residuals and its variants are used to solve the resulting equation.

- A recent approach to numerically integrate the Navier-Stokes equations directly as done by many computational fluid dynamics (CFD) simulations.

The first two approximate methods require a closure term for the heat release term. Closure formulas range from general closure terms relating velocity or pressure perturbations to heat release, to detailed flame models. These will be discussed later. The analysis then either takes the path of finding a dispersion relation for the frequency, via the Rankine-Hugoniot equations, or finding approximate solutions to the inhomogeneous partial differential equations (PDEs) using the Galerkin method. CFD simulations require large computational efforts if they are direct simulations, or need approximations for small turbulent scales as in large-eddy simulations. Examples from these approaches are discussed in more detail below.

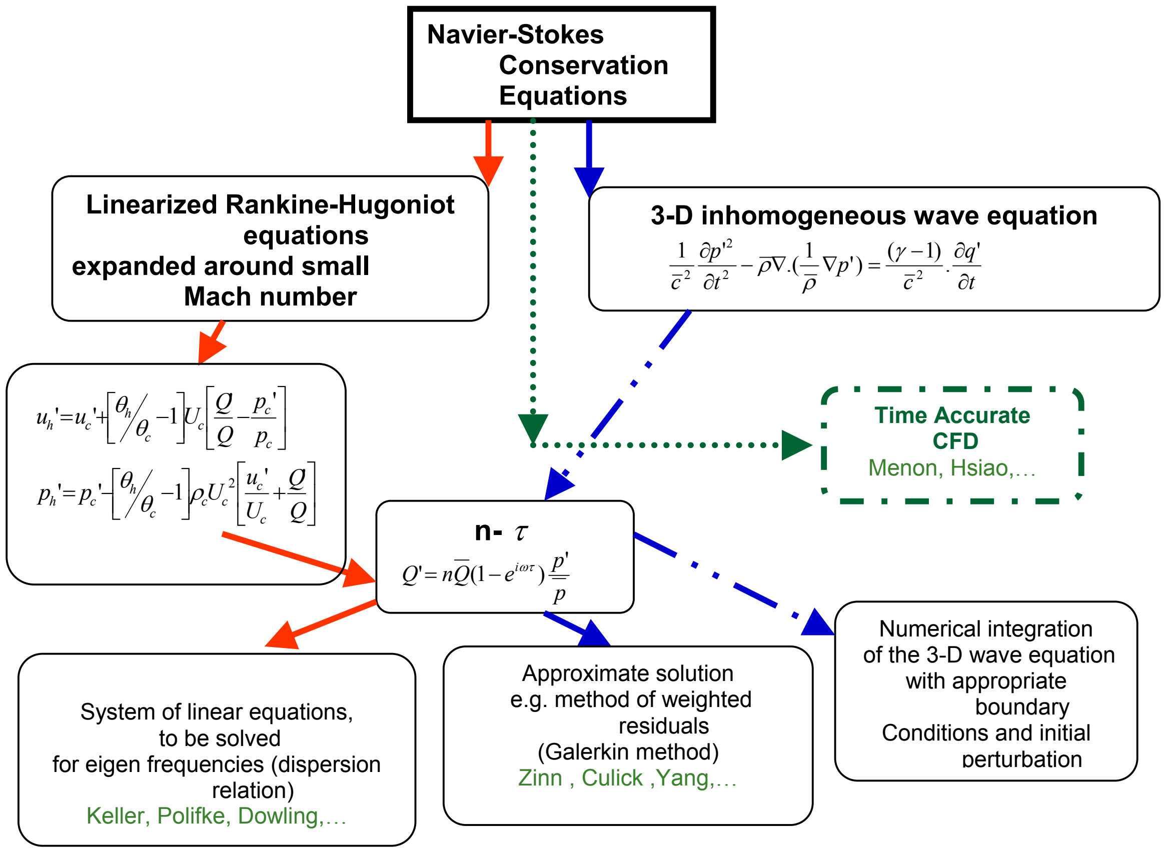

Figure 1: Summary of methods used to study combustion oscillatory instabilities

The (n- τ) Approach

Also referred to as the time-lag model, the n- τ approach evolved during research into liquid rocket instabilities by Crocco, Cheng and others [21]. The model provides a way to couple heat perturbations with flow-field perturbations. This is achieved by a pressure-interaction index, n, describing how the pressure oscillations affect combustion oscillations, and a time lag τ between the two fluctuations. The time lag is defined as the interval between the time when the pressure disturbance occurs at the flame to the time when heat is released in that location [6]. Crocco [21] assumed that the heat release is only affected by the pressure and is proportional to p^n, where n is the interaction index. These models have been used for linear and nonlinear perturbations alike [18,22] and are generally formulated to look like some variation of the expression,

$$Q' = n\overline{Q}(1 - e^{i\omega\tau})\frac{p'}{\overline{p}}, \tag{16}$$

where Q' is the perturbation of the heat release, p' is the pressure perturbation, and $\overline{Q}$ and $\overline{p}$ are the mean heat release rate and pressure respectively. The n-τ model in its original form is open to using empirical correlations between the heat and the pressure perturbation and in general should be viewed as a framework that may include both physical concepts and empirical observations.

Putnam provided a simple way of estimating the time lag for combustion instabilities such as those of interest here [6],

$$\tau = \frac{\delta}{V_p}, \tag{17}$$

where δ is the distance between the fuel injector to the flame front and V_p is the mean fuel velocity. This τ in essence is a mean convection time between the fuel port and the flame front. He later [6]modified the formula to account for an additional time lag based on observations from experimental results. The modified time-lag expression is

$$\tau = \frac{\delta}{V_p} + \left(\frac{a}{3}\right)\left(\frac{V_p}{S_L}\right)\left(\frac{1}{V_p}\right) = \frac{\delta}{V_p} + \frac{a}{3S_L}, \tag{18}$$

where a is the duct diameter and S_L is the laminar flame velocity. The additional time is intended to account of the transverse flame propagation across the diameter. There are various ways of interpreting the time lag. For example:

- The time lag appropriate for the liquid rocket applications often is thought to be associated with droplet evaporation and the consequent heat release, dependent on reaction rate. These processes are directly affected by the pressure, and hence Crocco determined that τ varies with p^n only.
- For the premixed combustion application considered here, the frequencies of concern are lower, and the time lag is more closely associated with the interval from the injection of reactants to their arrival at the flame front (convection time). This is closer to the expressions proposed by Putnam [6] in equations (17) and (18), which many of the studies on premixed combustor oscillations have adopted.

An Approach Employing the Rankine-Hugoniot Relations

Keller [23] and later Polifke [18] derived a set of algebraic correlations by expanding the Rankine-Hugoniot equations around the mean Mach number. The Rankine-Hugoniot relations are

$$\begin{aligned} \rho_c u_c &= \rho_h u_h \\ p_c + \rho_c u_c^2 &= p_h + \rho_h u_h^2 \\ h_c + \frac{1}{2} u_c^2 + q &= h_h + \frac{1}{2} u_h^2 \end{aligned} \quad , \qquad (19)$$

where ρ, u, p, h, and q are the density, velocity, pressure, enthalpy and heat addition respectively. The subscripts c and h represent the region prior to the flame (reactants/cold side) and the region after the flame (products/hot side). Keeping only the linear terms, they derived a relation between flow perturbations on both side of the flame, taken here to be a discontinuity with a temperature jump,

$$\begin{aligned} u_h' &= u_c' + \left[\theta_h \big/ \theta_c - 1 \right] U_c \left[\frac{Q'}{Q} - \frac{p_c'}{p_c} \right] \\ p_h' &= p_c' - \left[\theta_h \big/ \theta_c - 1 \right] \rho_c U_c^{\,2} \left[\frac{u_c'}{U_c} + \frac{Q'}{Q} \right] \end{aligned} , \qquad (20)$$

where u', p', θ, ρ, and U are the velocity perturbations, pressure perturbations, mean temperature, mean density and mean velocity, respectively.

Closure for the $\frac{Q'}{Q}$ term: To provide closure for the resulting algebraic equations, the term $\frac{Q'}{Q}$, representing the ratio of heat-release perturbations to mean heat release, was written in terms of pressure and/or velocity perturbations with a time lag τ. An example is the closure term for vortex shedding [18],

$$\frac{Q'}{Q} = \varepsilon e^{i\omega\tau}, \qquad (21)$$

where ε is interpreted as the percentage of reacting mixture entrained by the vortex. The model assumes τ to be a constant, which is a common assumption in the literature, along with the approximation that the interaction index n is constant. These approximations may not always be accurate, especially when there is significant flame movement, which would lead to a time varying τ, or different operating conditions, which would lead to a changing *n*.

An alternative to seeking an overall closure model for the heat-perturbation term $\frac{Q'}{Q}$ is to consider a flame model that incorporates some of the nonlinear effects involving the flame location, shape, area and response to flow-field perturbations. The flame model would affect the rate of heat release, since the total heat release can be related to the flame area by the expression

$$Q = C_p(\theta_h - \theta_c)\rho_c S_L A_{flame} , \qquad (22)$$

where C_p , ρ_c, S_L, and A_{flame} are the specific heat at constant pressure, density, laminar flame speed, and flame area respectively. In this thin-flame model, the resulting acoustic perturbations can be represented in terms of flame-area perturbations [24].

Efforts have been made to break the heat-perturbation term Q into several contributing components in other ways such as[11,25],

$$\dot{Q}_{total} = \dot{Q}_{\Delta H_r} + \dot{Q}_{S_L} + \dot{Q}_{A_{flame}} \qquad (23)$$

where $\dot{Q}_{\Delta H_r}$ is the perturbation resulting from heat-release fluctuations, $\dot{Q}_{S_L}$ is the perturbation caused by variation of the laminar flame speed and $\dot{Q}_{A_{flame}}$ represents fluctuations caused by instantaneous flame-area perturbations. These studies have assumed the flame to be a discontinuity separating reactants from products. In the work of Lieuwen [11], this flame front was tracked using a flame-tracking equation. Transfer functions were assigned to each term and contributions from these transfer functions were then incorporated into the combustion instability model. It was indicated that although the transfer functions were adequate approximations for the regime of small Strouhal number $St << 1$, at larger Strouhal numbers there were reported differences between experiment and theory [11]. In these studies, the Strouhal number is defined as

$$S_t = \frac{\omega R}{\overline{S_{L0}}}, \tag{24}$$

where ω is the frequency, R is the radius of the combustor inlet, $\overline{S_{L0}}$ is the mean laminar flame speed at a reference point (e.g. center point).

Stability Studies: In studying the stability of combustion systems, the pressure and velocity perturbations were taken to behave in a harmonic manner, similar to equation (2). Polifke et al. [26] constructed a network model for a simplified combustion system, in which each component of the system was represented by an acoustic impedance. Assuming negligible contribution from the mean flow (low Mach number approximation), and starting from a fuel plenum with boundary conditions set to zero (no flow perturbations), the one-dimensional coupled system of equations developed in this process accounted for pressure losses and velocity changes caused by duct friction and area changes up to the flame location. Jump conditions were introduced at the flame location as the flame was assumed to be a discontinuity of negligible thickness resulting in an instantaneous temperature jump. The combustor zone after the flame was approximated by a constant-area duct and a choked flow conditions were taken at the combustor exit, Mach number equal to unity, representing maximum mass flow and permitting no flow perturbations to propagate upstream from the turbine. A small harmonic perturbation was then numerically introduced at the plenum exit. The coupled system of equations was manipulated to derive a dispersion equation in the frequency ω. The roots of the equation are the complex eigenfrequencies [18,27].

A stability criterion was determined by looking at the cycle increment, defined [18] as the percentage by which an infinitesimal amplitude may grow in one cycle,

$$CI(\omega) = e^{\left[-2\pi\frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)}\right]} - 1\,. \tag{25}$$

Here $CI(\omega)$, the cycle increment for a particular frequency, is an exponential function of the ratio of the imaginary to real part of the eigenfrequency and is a measure of the growth rate of the amplitude, and negative values of $CI(\omega)$ indicate decay.

As the system of equations grows larger, (adding more mechanisms or including more ducting upstream or down stream from the flame), finding all the eigenfrequencies of the dispersion relation becomes a harder numerical task. Also, the dispersion relations obtained by the above studies do not provide concrete information about frequencies other than the eigenfrequencies, and whether they will grow or damp. As operating conditions of the combustor change and the energy distribution shifts between modes, some of these undetermined eigenfrequencies may be susceptible to growth as well.

An alternative to seeking eigenfrequencies and dispersion relations is the approach of inserting a closure term for the heat-release perturbations term in equations (20). For example, putting $\frac{Q'}{Q} = \varepsilon e^{i\omega\tau}$ with, $\varepsilon = 0.05$ (5% of the reacting mixture is entrained by the vortex), enables pressure and velocity perturbations' amplitudes to be calculated. Inspection of the resulting pressure-perturbation amplitude in time (growth or decay) provides a way of inferring growth or decay information from the time history of the perturbation [18].

In this method the choice of the value of ε is empirical, and it is viewed as the only nonlinear addition to the linearized equations. It then becomes the driving source for possible amplification. This makes predicting instability using this method wholly dependent on the choice of the driving term, presumed to arise from nonlinear phenomena in a formulation which nonetheless is linear.

Use of Green's Functions

An intermediate level of analysis, between the linearized Rankine-Hugoniot equations and a full time-accurate CFD simulation, is the use of approximate methods to solve the inhomogeneous PDE. These approximate solutions attempt to capture more of the physics than simple closure approaches can capture. Starting from the linearized Navier-Stokes conservation equations and ignoring the mean flow (Mach number very small so it can be approximated as being equal to zero), Hegde et al. [28] formulated the problem in the form of an inhomogeneous wave equation for the pressure perturbation,

$$\frac{1}{\bar{c}^2}\frac{\partial p'^2}{\partial t^2} - \bar{\rho}\nabla\cdot\left(\frac{1}{\bar{\rho}}\nabla p'\right) = \frac{(\gamma-1)}{\bar{c}^2}\frac{\partial q'}{\partial t}. \qquad (26)$$

To solve the problem, a Green's function was used to represent the heat perturbation term in equation (26). Overall, their use of this method provided some insight but fell short of yielding a conclusive answer because of their omission of a formal coupling between the heat release and flow-field perturbations [9].

In the case of a formal coupling between the pressure perturbations and the heat perturbations, the use of a Green's function essentially simplifies the inhomogeneous PDE into an integral equation to be solved iteratively.

Approaches making use of a Galerkin Expansion

The approach is a variation of the method of weighted residuals developed for solving PDEs. Powell and Zinn [29] applied a linearized Galerkin technique in their investigation of axial and transverse instabilities in liquid rocket motors. A similar approach was independently developed and used to analyze solid rockets instabilities by Culick [30].

Again the approach is from the inhomogeneous wave equation for the pressure perturbations p',

$$\nabla^2 p' - \frac{\partial^2 p'}{\partial t^2} = h\,, \tag{27}$$

where the source term h is the contribution of the heat perturbations to the pressure perturbations. Using the notation of Culick [31] this term can be written as,

$$\begin{aligned} h = &-\bar{\rho}\nabla^2 \cdot \left(\bar{u} \cdot \nabla u' + u' \cdot \nabla \bar{u}\right) + \frac{1}{\bar{c}^2}\bar{u} \cdot \nabla \frac{\partial p'}{\partial t} + \frac{\bar{\gamma}}{\bar{c}^2}\frac{\partial p'}{\partial t}\nabla \cdot \bar{u} - \nabla \cdot \left[\overline{\rho\left(u' \cdot \nabla\right)u' + \rho\frac{\partial u'}{\partial t}}\right] \\ &+ \frac{1}{\bar{c}^2}\frac{\partial}{\partial t}\left(u' \cdot \nabla p'\right) + \frac{\bar{\gamma}}{\bar{c}^2}\frac{\partial p'}{\partial t}\nabla \cdot u' + \nabla \cdot F' + \frac{1}{\bar{c}^2}\frac{\partial p'}{\partial t} \end{aligned}\,. \tag{28}$$

The boundary condition is

$$n \cdot \nabla p' = -f\,, \tag{29}$$

In which $n \cdot \nabla p'$ is zero for rigid-wall boundary conditions, and *f* is the effect of the heat addition.

The solution for equation (27) was approximated by a series expansion in the natural modes of the combustion chamber, which are the solutions of the wave equation with no heat perturbation and rigid-wall boundaries. The solution was given the form:

$$p'\left(r,t\right) = \sum_m \eta_m(t)\psi_m(r)\,, \tag{30}$$

where $\eta_m(t)$ are the amplitudes of the pressure perturbations and ψ_m are the acoustic eigenfunctions; these are the geometric natural modes with no flow or heat addition. Given the mode shape, the use of the Galerkin method tracks in time whether the amplitude of each mode m $\eta_m(t)$ will grow or decay.

The Galerkin methodology has been used to look at various longitudinal and transverse modes and has been used for both linear and nonlinear analyses. It has been observed that different modes may or may not grow depending on the number of expansion terms included in the Galerkin approximation [32]. Inclusion of more modal terms from the expansion results in more effort and eventually to a larger nonlinear system of ordinary differential equations. Finally, detailed knowledge of the mode shapes and therefore of the combustor geometry is needed to be able to proceed with this type of analysis.

Unsteady Computational Fluid Dynamics Simulation (CFD)

Direct modeling of combustion processes in general requires extensive computational resources. In addition to solving for the flow field and chemistry, an inherently stiff system of equations, direct solution requires solving for the transport properties of *N* species produced or consumed during the reaction processes. Unsteady CFD, though far superior to previous approximate methods in capturing details of the flow field, requires extensive time and computational resources to model the complex flow field of multi species, dimension, and time represented in evolving Navier-Stokes equations.

Instead of computing fine-scale fluid details directly with direct numerical simulation (DNS), the practice has been to limit calculations to large fluid structures, thus reducing the computational requirements. A separate, simple model represents the fine structures and quantifies only their contribution to the flow field. This treatment, referred to as large-eddy simulation (LES), utilizes a subgrid model to indirectly account for the fine-scale turbulent structures, which are assumed to be homogeneous and possess a universal character.

Hsiao et al. [33] studied the instabilities in a 3-D model of an annular combustor using LES. He later used CFD modeling results to incorporate into a low order acoustic model for predicting oscillatory instabilities. Stone and Menon [34] studied the effect of swirl on the stability of premixed combustors, again employing LES, they effectively captured the contribution of large structures to mass, momentum, and energy "vortex structures," while empirical and or analytic formulas were used for the smaller turbulent structures. Comparisons for various fuel/air ratios showed their effect on imperfect mixing and the result on the combustor instability.

Steele et al. [1] used an LES code to run time-accurate simulations of a model combustor with the approximate dimensions of one of the Solar Turbines Mars combustor family. They predicted a range of instabilities that later were compared relatively well with experimental results. They also used an (n-τ) model to look at the stability characteristics of that particular combustor. This effort, using results from CFD along with a low-order model, resulted in what some in the industry term the (τ-F) criteria, which is essentially a convective time lag multiplied by the frequency of interest. The work enabled them to identify some of the stable/unstable operating regimes for gas turbine engines and served as a quick design guideline to avoiding zones of instability, but never explained why instabilities would or would not occur.

To date much of the unsteady CFD literature has focused on simple geometries. In many cases results from steady CFD have been used to compliment low order models.

Summary of Reviewed Studies

Much of the analysis in the literature has assumed the wave behaves as a one-dimensional acoustic wave. Very little analysis has been done on multi-dimensional acoustic wave propagations and the effects of reflecting and or refracting waves. Lieuwen [35] investigated the acoustic field in the near field of a flame using 2-D flame zone geometry and a boundary element method code. He found that while the pressure field remained almost one-dimensional with very small departures, the velocity field behaved weakly 2-dimensionally at the center of the flame (variation from the 1-D wave approximation increasing slightly with the frequency) returning monotonically to the one-dimensional behavior closer to the wall.

The previous numerical and experimental research has focused on simple geometries such as straight ducts or constant area combustors, and in some cases has used network models, a method by which a complex geometry is broken into a series of different size ducts to compensate for different diameters and area changes [18,27]. This has allowed the simplification of the task of calculating the natural modes and convection times appropriate to the geometry.

Neglecting the mean flow is a common simplification of the problem adopted by all of the approximate methods. Mean flow contribution comes in two types, a velocity contribution to the acoustic wave and additional waves such as vortex shedding and entropy waves [9]. Dowling [36] studied the impact of the mean flow on combustion oscillations and concluded that mean flow effects are negligible for Mach numbers less than 0.2 (many practical combustors), while for higher Mach number flows, the presence of the mean flow adds the possibility of new mode oscillations.

Motivation for a New Approach

Previous research efforts have attempted to either identify the growth or decay of a particular mode, generally the natural modes of the geometry as in the Galerkin approach, or have attempted to identify the modes that will grow or decay by solving a dispersion relation, as in the time lag approaches. The susceptibility of particular modes to growth or decay depends on the selection of amplification and/or damping mechanisms included in the governing equations.

The authors couldn't find much prior work approaching the instability issue with a direct acoustic energy conservation approach. This approach allows the flexibility to calculate contributions from amplification and/or attenuation across all frequencies, rather than predicting eigenmodes. It is our belief that a direct acoustic energy approach could be of tremendous benefit in determining the susceptibility of all frequencies to growth or decay as operating conditions in a gas turbine change. This approach may also give designers a better handle on improving passive damping in the initial design phase, as they would have the ability to scan various frequencies and investigate their susceptibility to oscillatory instability at different operating conditions. We expect that this effort will provide a new prospective on and enhanced prediction of combustion oscillatory instabilities.

Investigation Through Acoustic Energy Conservation Equation

It is clear from previous research that the occurrence of an oscillatory instability in a combustor follows a series of events resulting in a rapid increase in the acoustic energy content inside the control volume. Therefore, the goal in this modeling effort is a careful accounting of the sources and sinks for acoustic energy, to identify conditions of instability.

The analysis begins with the unsteady Navier-Stokes equations. Following the derivation by Williams [2], the linearized perturbation equations render an expression for the instantaneous acoustic energy in the form:

$$e = \frac{p'^2}{2\bar{P}\gamma} + \frac{\bar{\rho}v'^2}{2}\,. \tag{31}$$

Consequently, the conservation of the acoustic energy in a combustion chamber can be written as:

$$\frac{\partial e}{\partial t} + < PV > .(\rho' v) + \nabla(eV) + \Phi = 0\,, \tag{32}$$

where Φ is a source term for amplification/damping to the acoustic energy [2].

Employing the divergence theorem, the above expression can be written as follows:

$$\frac{d}{dt}\iiint_V edV = -\oiint_A Pp'v'.ndA - \oiint_A e\bar{v}.ndA + \iiint_V \phi dV\,, \tag{33}$$

where in this equation the first term on right side is the contribution by the boundary work, the second is the acoustic energy convected by the mean flow, and the third term is the combined amplification/damping effects from different mechanisms that contribute to combustion oscillations.

Linear Analysis

The mechanisms attributing to amplification and damping of acoustic energy are to be assessed independently to find their amplification coefficient α, a reciprocal time (or a rate), which can be computed as follows:

$$\alpha = \left[\frac{-\int_A p'v'.ndA - \int_A e\bar{v}.ndA + \int_V \Phi dV}{2\bar{e}V} \right], \qquad (34)$$

where

$$\bar{e} = \frac{1}{V}\int_V edV , \qquad (35)$$

is the average acoustic energy per cycle in the combustor volume.

The acoustic energy at any time can be calculated from the expression

$$e = e_o^{\alpha t} . \qquad (36)$$

In the linear analysis

$$\alpha = \sum \alpha_{amp} - \sum \alpha_{damp} , \qquad (37)$$

enables the inclusion of all contributing factors to an oscillation event in a simple algebraic manner.

Stability for each frequency stability criteria can be assessed by looking at:

$$S_{sf} = \frac{\sum \alpha_{amp}}{\sum \alpha_{damp.}}, \qquad (38)$$

where $S_{sf} > 1$ indicates that amplification overcomes damping and that an oscillation could occur at the conditions and frequency examined.

We believe that the linear analysis tool proposed here would be useful at the initial design phase, allowing for rapid analysis of various designs before moving onto a complex nonlinear study, and/or a costly CFD simulation of the problem. By independently evaluating the effects of amplification and driving mechanisms, a linear analysis tool can also be useful for understanding likely root causes and potential solutions for operational problems.

Attach: *Progress presentation made at Solar Turbines, April, 5th, 2005*

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